A Rheo-Geotechnical Approach to Penetration Testing

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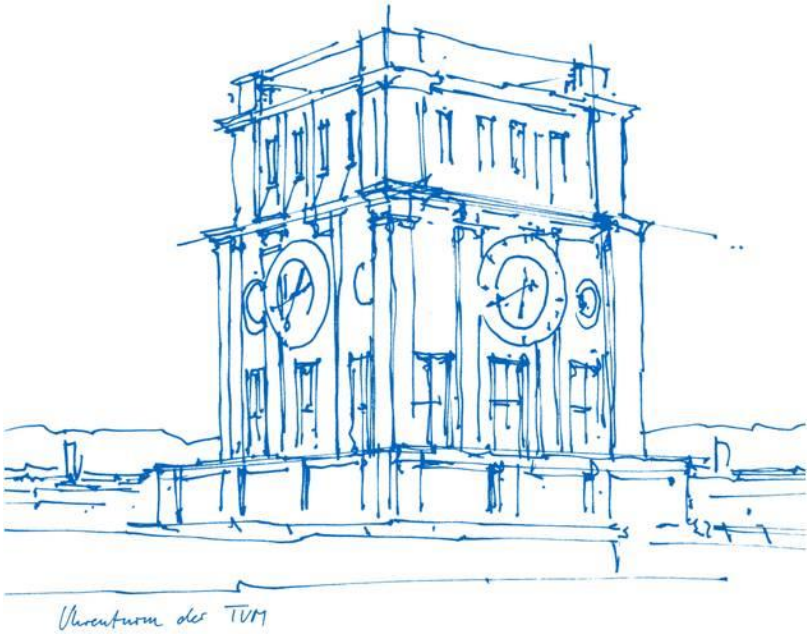
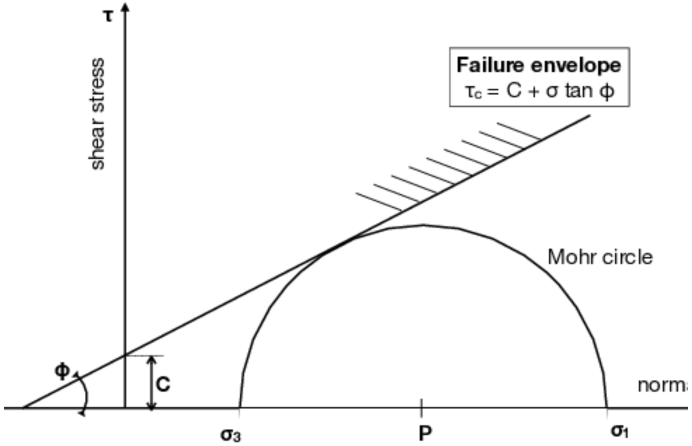


Context – Soil Mechanics

Collapse of a grain elevator at Transcona, Manitoba, Canada in 1913 is a famous example of a geotechnical failure

Context – Soil Mechanics

Mohr-Culomb Parameters



Ultimate Bearing-Capacity

$$q = \underbrace{S_c\, c\, N_c(\varphi)}_{\substack{\textbf{Cohesive Term} \\ \text{(Material Strength)}}} + \underbrace{S_\gamma\, \gamma D\, N_\gamma(\varphi)}_{\substack{\textbf{Unit Weight Term} \\ \text{(Lateral/Width Effect)}}} + \underbrace{S_q\, \gamma z\, N_q(\varphi)}_{\substack{\textbf{Surcharge Term} \\ \text{(Depth Dependent)}}}$$

c	Cohesion [kPa]
φ	Internal Friction Angle [°]
D	Punch Diameter [mm]
z	Depth [mm]
γ	Unit Weight [kN/m^3]
S_c, S_γ, S_q	Shape Factors (Geometry Dependent)
N_c, N_γ, N_q	Bearing Capacity Factors (Function of φ)

Theoretical Framework: Requirements and Assumptions

1. Material State

- Homogeneous & Isotropic
- Constant Properties
- Incompressible Behavior

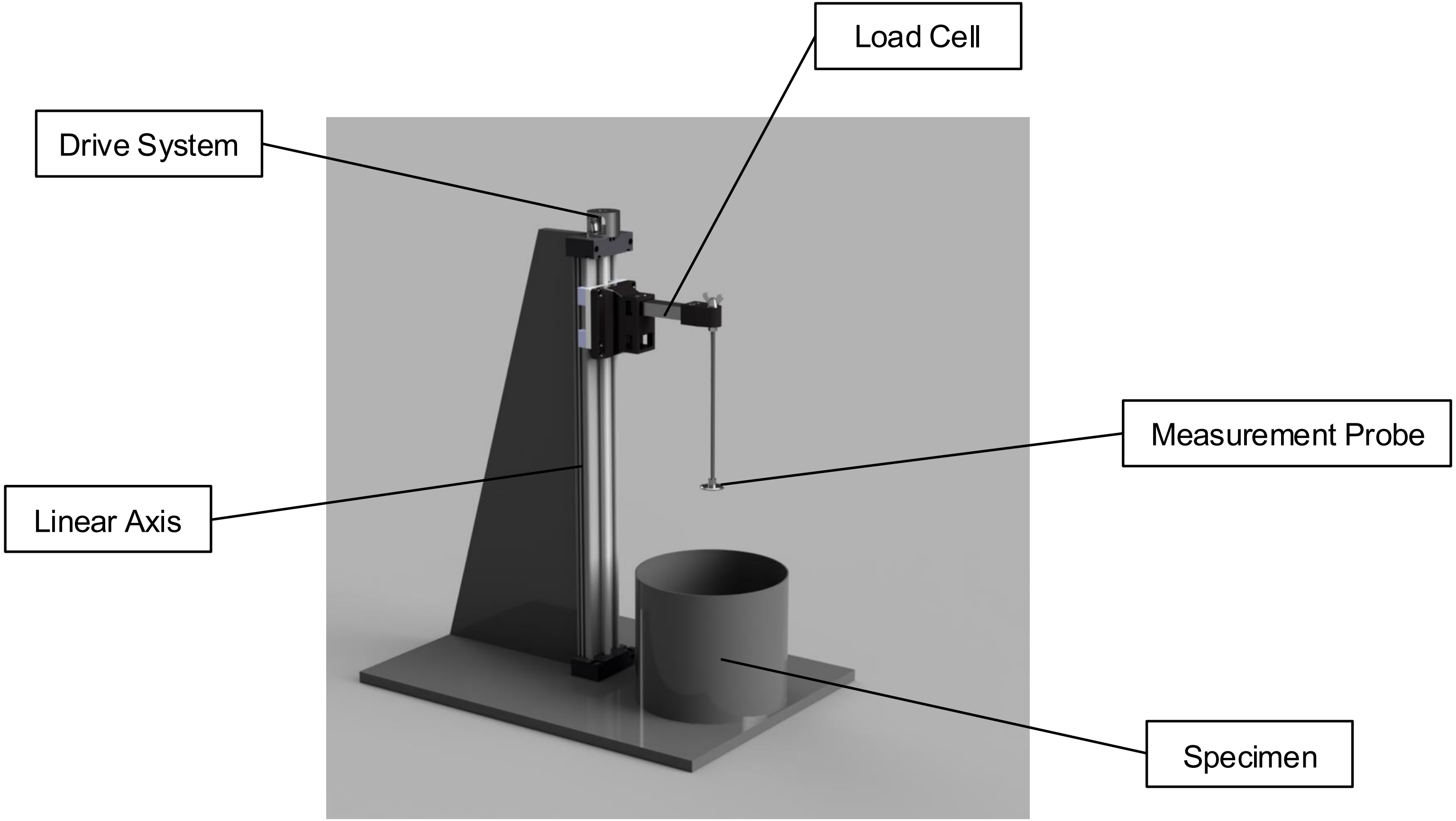
2. Geometry & Boundaries

- Semi-infinite Half-space
- Open Shaft Cavity
- Axisymmetric Loading

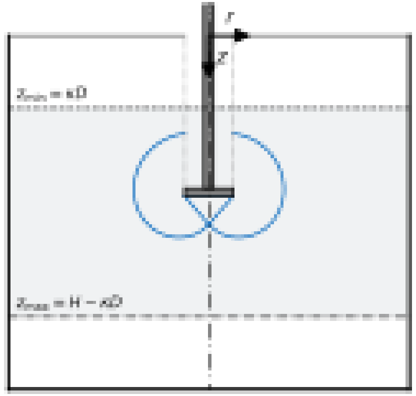
3. Mechanical Regime

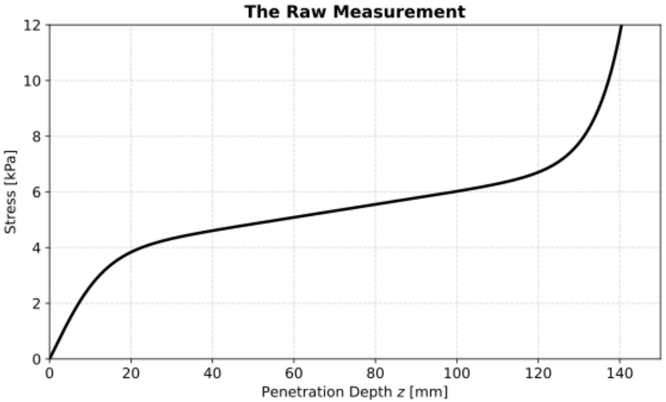
- Quasi-static Loading
- Prandtl Failure Mechanism
- Literature Shape Factors

Experimental Setup



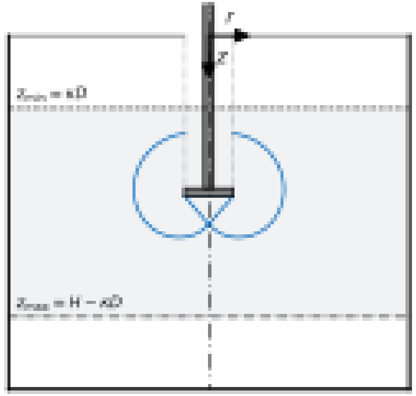
Theoretical Framework

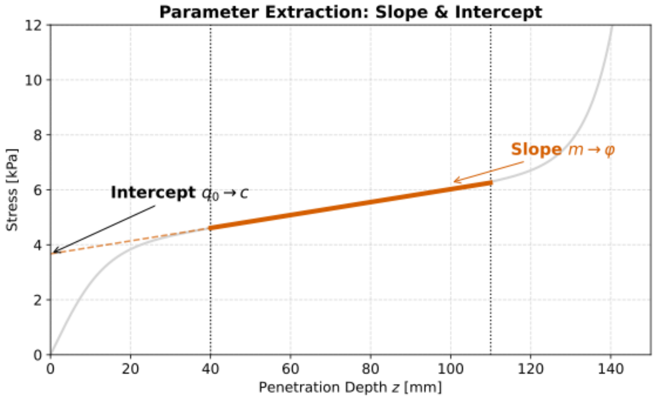




$$q(z) = \underbrace{S_c\, c\, N_c(\varphi)}_{\substack{\textbf{Cohesive Term} \\ \text{(Material Strength)}}} + \underbrace{S_\gamma\, \gamma D\, N_\gamma(\varphi)}_{\substack{\textbf{Unit Weight Term} \\ \text{(Lateral/Width Effect)}}} + \underbrace{S_q\, \gamma z\, N_q(\varphi)}_{\substack{\textbf{Surcharge Term} \\ \text{(Depth Dependent)}}}$$

Theoretical Framework




Parameter Extraction: Slope & Intercept

$$q(z) = \underbrace{S_c\, c\, N_c(\varphi)}_{\substack{\textbf{Cohesive Term}\\ \text{(Material Strength)}}} + \underbrace{S_\gamma\, \gamma D\, N_\gamma(\varphi)}_{\substack{\textbf{Unit Weight Term}\\ \text{(Lateral/Width Effect)}}} + \underbrace{S_q\, \gamma z\, N_q(\varphi)}_{\substack{\textbf{Surcharge Term}\\ \text{(Depth Dependent)}}}$$

$Step\ 1\!: Slope \rightarrow Internal\ Friction\ Angle\ \varphi$

$$\frac{dq(z)}{dz} = m = S_q\, \gamma\, N_q(\varphi)$$

$$\Rightarrow \quad \varphi = N_q^{-1}\left(\frac{m}{S_q\, \gamma}\right)$$

$Step\ 2\!: Intercept \rightarrow Cohision\ c$

$$\bar{q}_{\text{net}} = \overline{q(z) - S_q\, \gamma\, z\, N_q(\varphi)}$$

$$\Rightarrow \quad c = \frac{\bar{q}_{\text{net}} - S_\gamma\, \gamma\, D\, N_\gamma(\varphi)}{S_c\, N_c(\varphi)}$$

Experimental Program: Geometry & Rate dependency

Phase 1: Geometry

D20_V5

D25_V5

D30_V5
(Reference)

D35_V5

D40_V5

Phase 2: Rate

D30_V1

D30_V5
(Reference)

D30_V10

D30_V50

D30_V150

D30_V300

D30_V600

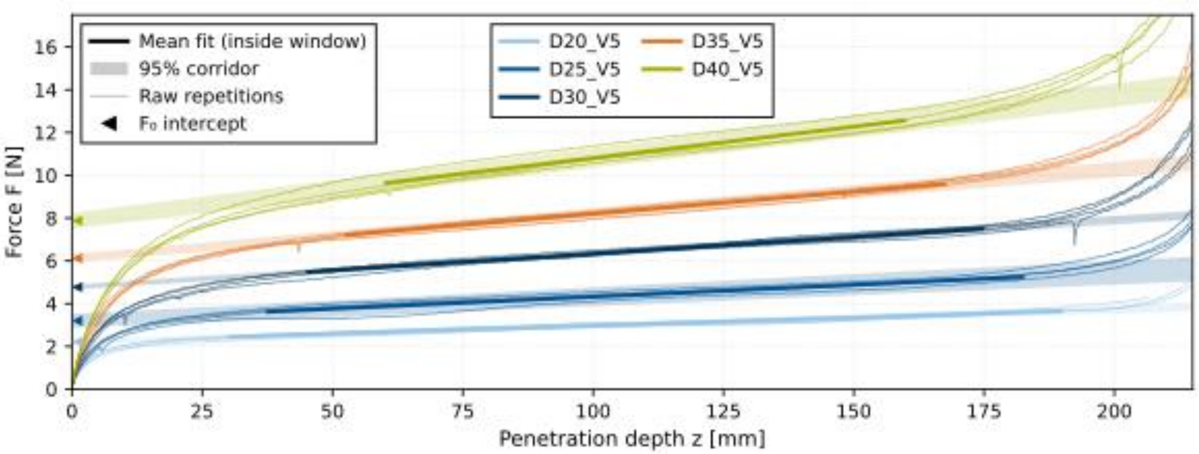
Study Objectives

- **Phase 1: Prove static theory validity**
  → Benchmark against DSB & Vane
- **Phase 2: Quantify viscous rate effects**
  → Benchmark against Vane

Material: Montmorillonite-Glycerin
(Inert Model Paste, 1:1.1 Ratio)

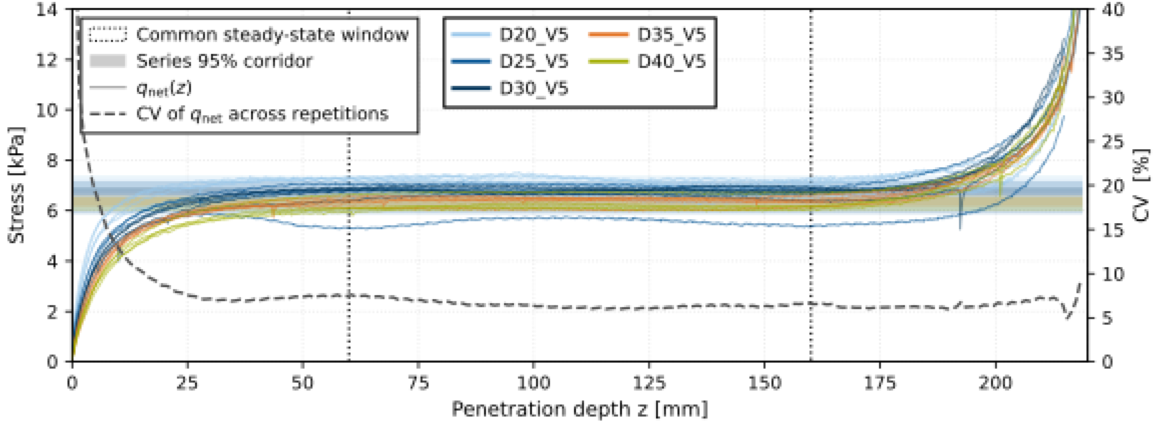
Nomenclature: DXX_VYY
D = Punch Diameter [mm]
→ 2-fold variation (20 - 40 mm)
V = Penetration Velocity [mm/min]
↓ Over 2 orders of magnitude (1 - 600)

Phase 1 – Results: Penetration Testing / Raw Data



- **n = 4 per diameter** (total **20** penetration tests)
- Quasi-steady window supports linear fit
- Raw Data: Diameter dependent

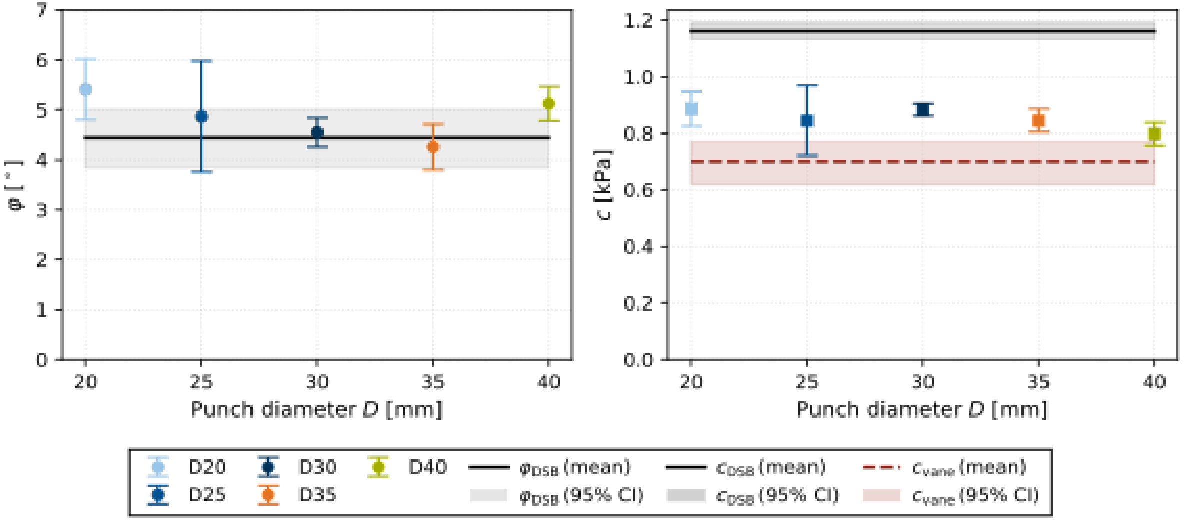
Phase 1 – Results: Penetration Testing

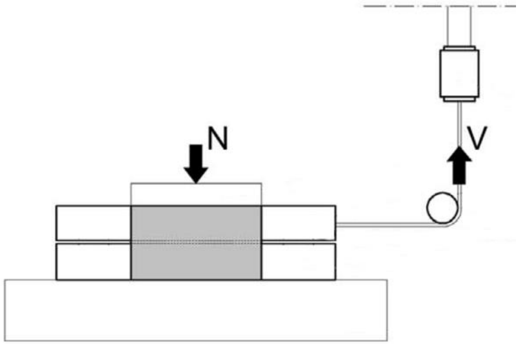


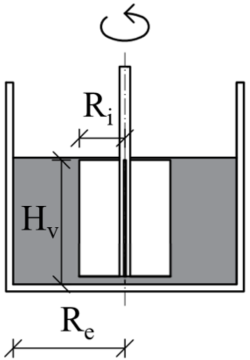
- $q_{net}(z)$ collapses to a **common plateau**
- **Common steady-state window** across all diameters
- Repeatability: **low CV** ($\approx 5\%$) in the plateau region
- φ ≈ 4.85° (CV 9.4%), c ≈ 0.85 kPa (CV = 4.8%)

➢ **No geometry dependence** in this range (curves agree)

Phase 1 – Results: External Benchmark







Inner Friction Angle φ:	Direct Shear Box	4.45°
	Penetration	4.85°
Cohesion c:	Direct Shear Box	1.16 kPa
	Penetration	0.85 kPa
	Vane	0.70 kPa

Experimental Program: Geometry & Rate dependency

Phase 1: Geometry

D20_V5
D25_V5
D30_V5 (Reference)
D35_V5
D40_V5

Phase 2: Rate

D30_V1
D30_V5 (Reference)
D30_V10
D30_V50
D30_V150
D30_V300
D30_V600

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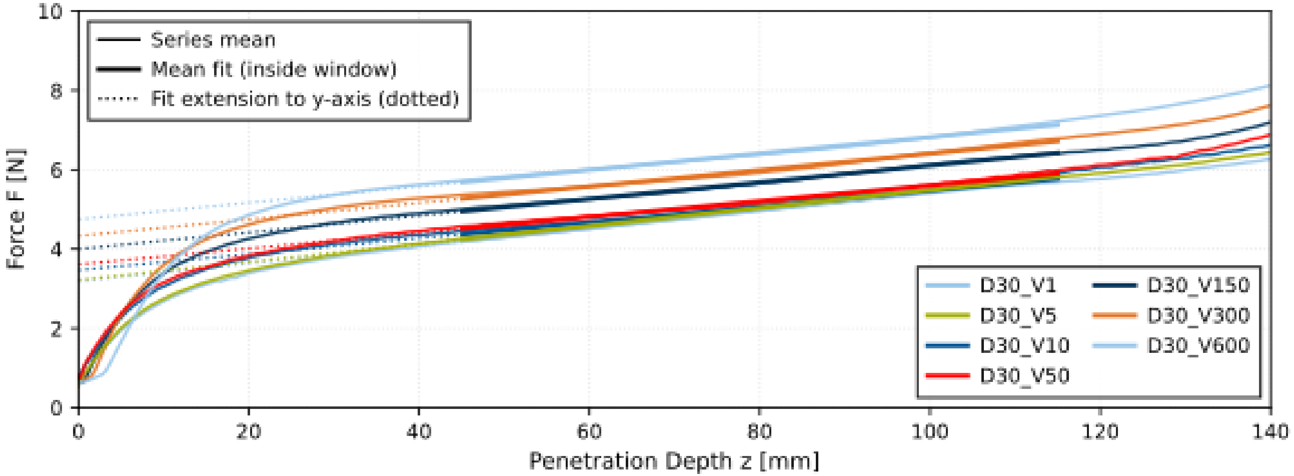
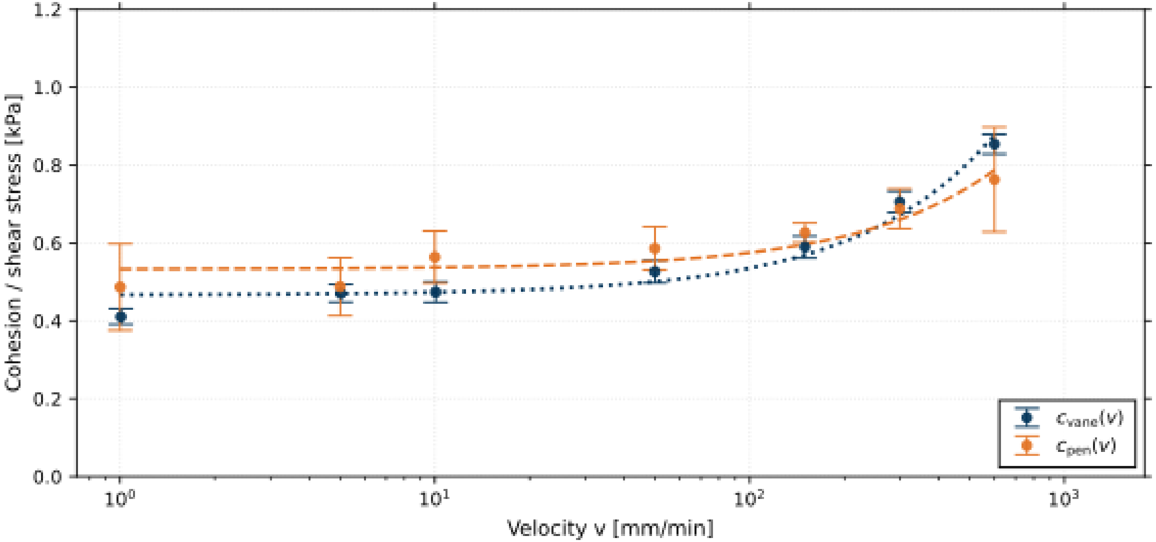
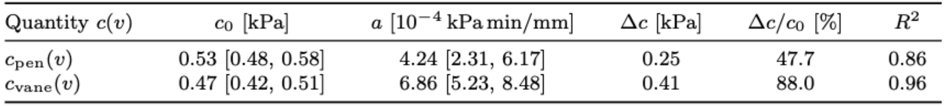
Phase 2 – Results: Penetration Testing



- **V = 1–600 mm/min**, **n = 4** per series
- Curves ≈ **parallel** in steady window
- Rate effect mainly in **c (0.49 - 76 kPa)**,
- **φ (~7.77°)**

Phase 2 – Results: Penetration testing





Quantity $c(v)$	c_0 [kPa]	a [10^{-4} kPa min/mm]	Δc [kPa]	$\Delta c/c_0$ [%]	R^2
$c_{\text{pen}}(v)$	0.53 [0.48, 0.58]	4.24 [2.31, 6.17]	0.25	47.7	0.86
$c_{\text{vane}}(v)$	0.47 [0.42, 0.51]	6.86 [5.23, 8.48]	0.41	88.0	0.96

- **Tangential vane speed** as kinematic proxy
- Similar kinetic trends
- Close c_0 projections

Conclusions & Outlook

Conclusions

Penetration + bearing-capacity inversion gives **apparent** c, φ from one test

Phase 1: no diameter trend (20 – 40 mm), benchmark-consistent

Phase 2: rate mainly increases **apparent** c; phi ~ stable; trend consistent with vane

Outlook

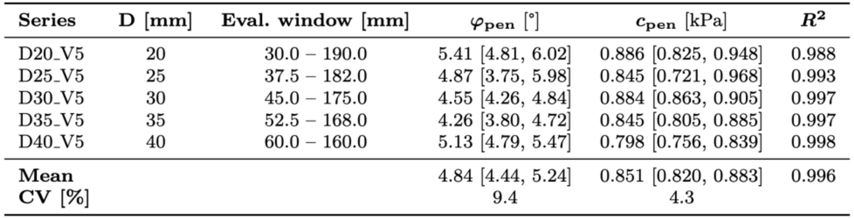
Extend to **time-dependent, aggregate-rich mixes** and early-age evolution c(t), φ(t)

Improve calibration: **shape factors**, boundary effects, kinematic scaling vs. rheometry

Link to buildability: **stability models** and process control windows

Backup

Phase 1: Geometry



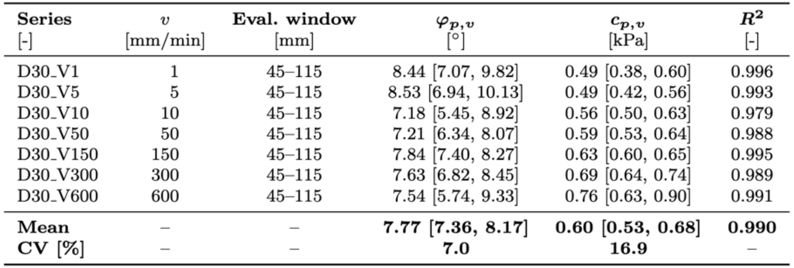
Series	D [mm]	Eval. window [mm]	φ_{pen} [°]	c_{pen} [kPa]	R^2
D20_V5	20	30.0 – 190.0	5.41 [4.81, 6.02]	0.886 [0.825, 0.948]	0.988
D25_V5	25	37.5 – 182.0	4.87 [3.75, 5.98]	0.845 [0.721, 0.968]	0.993
D30_V5	30	45.0 – 175.0	4.55 [4.26, 4.84]	0.884 [0.863, 0.905]	0.997
D35_V5	35	52.5 – 168.0	4.26 [3.80, 4.72]	0.845 [0.805, 0.885]	0.997
D40_V5	40	60.0 – 160.0	5.13 [4.79, 5.47]	0.798 [0.756, 0.839]	0.998
Mean			4.84 [4.44, 5.24]	0.851 [0.820, 0.883]	0.996
CV [%]			9.4	4.3	

$$N_q(\varphi) = e^{\pi \tan\varphi} \tan^2\left(45^\circ + \frac{\varphi}{2}\right)$$

$$N_c(\varphi) = \frac{N_q(\varphi) - 1}{\tan\varphi}$$

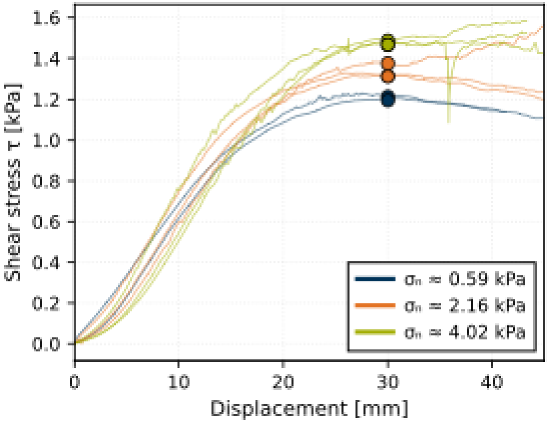
$$N_\gamma(\varphi) = (N_q(\varphi) - 1)\tan(1.4\varphi)$$

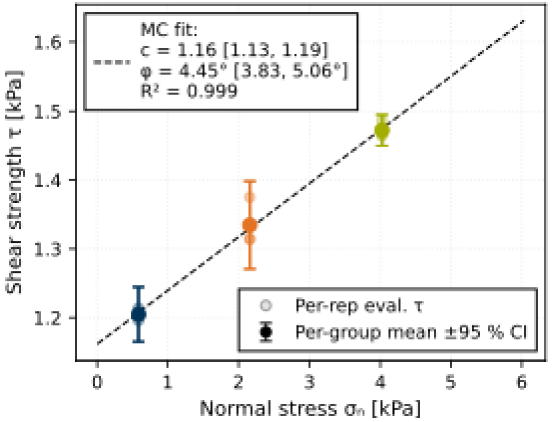
Phase 2: Rate

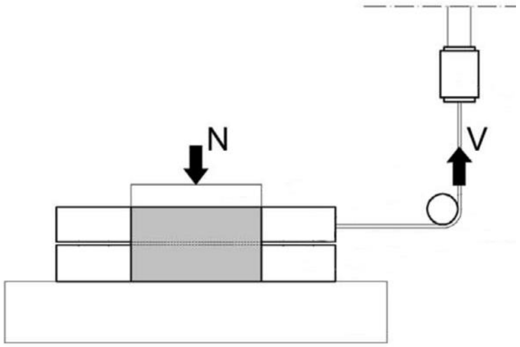


Series [-]	v [mm/min]	Eval. window [mm]	$\varphi_{p,v}$ [°]	$c_{p,v}$ [kPa]	R^2 [-]
D30_V1	1	45–115	8.44 [7.07, 9.82]	0.49 [0.38, 0.60]	0.996
D30_V5	5	45–115	8.53 [6.94, 10.13]	0.49 [0.42, 0.56]	0.993
D30_V10	10	45–115	7.18 [5.45, 8.92]	0.56 [0.50, 0.63]	0.979
D30_V50	50	45–115	7.21 [6.34, 8.07]	0.59 [0.53, 0.64]	0.988
D30_V150	150	45–115	7.84 [7.40, 8.27]	0.63 [0.60, 0.65]	0.995
D30_V300	300	45–115	7.63 [6.82, 8.45]	0.69 [0.64, 0.74]	0.989
D30_V600	600	45–115	7.54 [5.74, 9.33]	0.76 [0.63, 0.90]	0.991
Mean	–	–	**7.77 [7.36, 8.17]**	**0.60 [0.53, 0.68]**	**0.990**
CV [%]	–	–	**7.0**	**16.9**	–

Backup - DSB





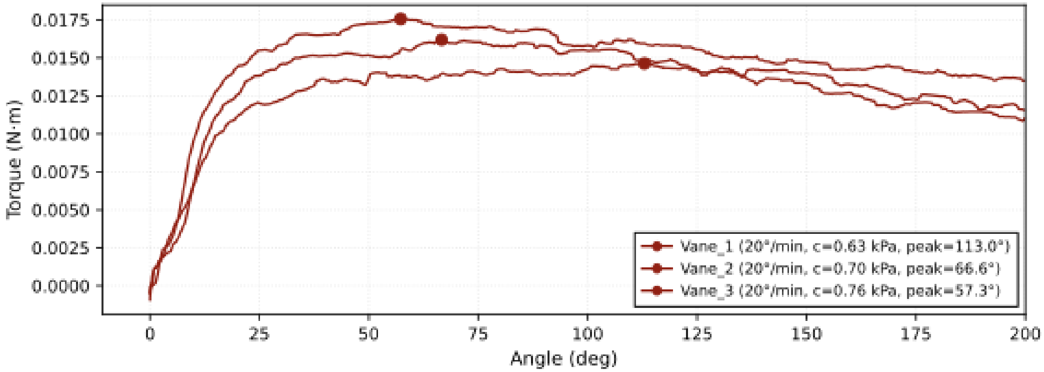


Benchmark: Direct Shear Box
(Standard: Modified DIN EN ISO 17892-10)

- Rate: 1.2 mm/min
- Validates: Friction (φ) & Cohesion (c)

$$\tau_f = c + \sigma_n \tan\varphi$$

Backup



Benchmark: Vane Shear Test
(Standard: DIN EN ISO 22476-9)

- Geometry: D = 20 mm, H = 30 mm
- Rate: 20°/min
- Method: Peak torque evaluation
- Validates: Cohesion (c)

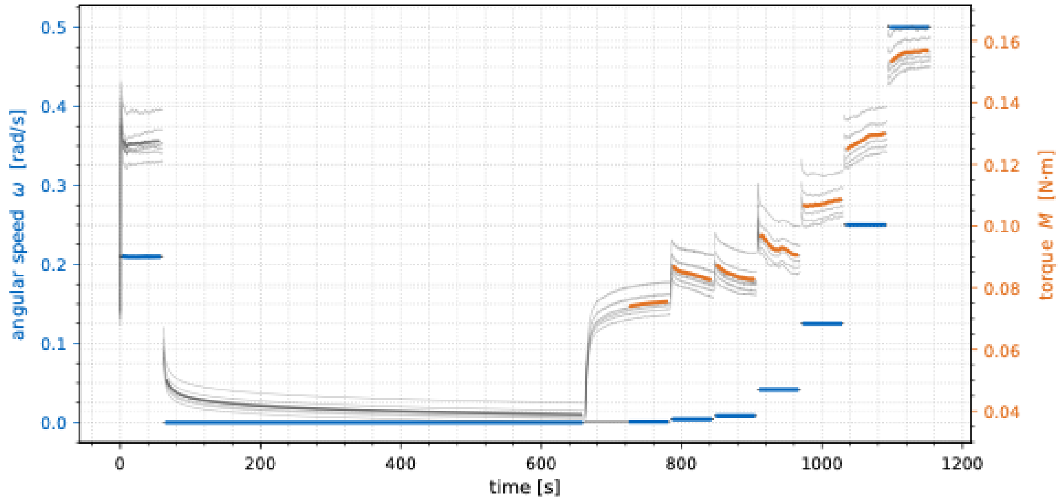
$$c_{vane} = \frac{2 \cdot 10^6 \cdot T_{max}}{\pi D_v^2 \left(H_v + \frac{D_v}{3}\right)}$$

Benchmark: Direct Shear Box
(Standard: Modified DIN EN ISO 17892-10)

- Rate: 1.2 mm/min
- Validates: Friction (φ) & Cohesion (c)

$$\tau_f = c + \sigma_n \tan \varphi$$

Backup



Step	v [mm/min]	T_{mean} [mNm]	c_{vane} [kPa]
1	1.01	74.89	0.41
2	5.03	84.37	0.47
3	10.06	84.13	0.47
4	50.01	92.96	0.53
5	149.92	107.31	0.59
6	299.96	128.13	0.71
7	600.04	156.03	0.85

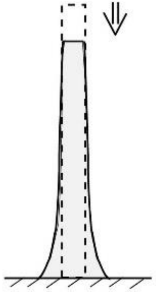
Benchmark: Vane Shear Test

(Standard: DIN EN ISO 22476-9)

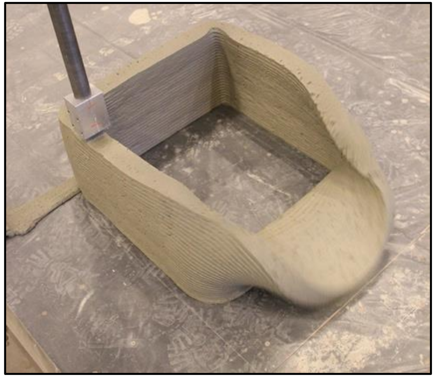
- Geometry: …..
- Rate: 20°/min
- Method: Peak torque evaluation
- Validates: Cohesion (c)

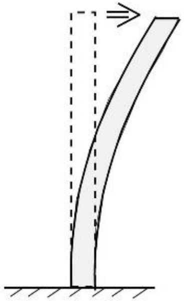
$$c_{vanr} = \frac{2 \cdot 10^6 \cdot T_{max}}{\pi D_v^2 \left(H_v + \frac{D_v}{3}\right)}$$

Context – Buildability Requires Material Strength Parameters

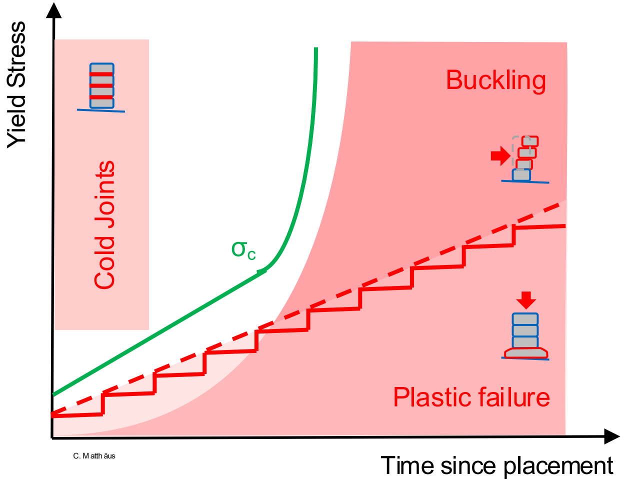


Plastic collapse





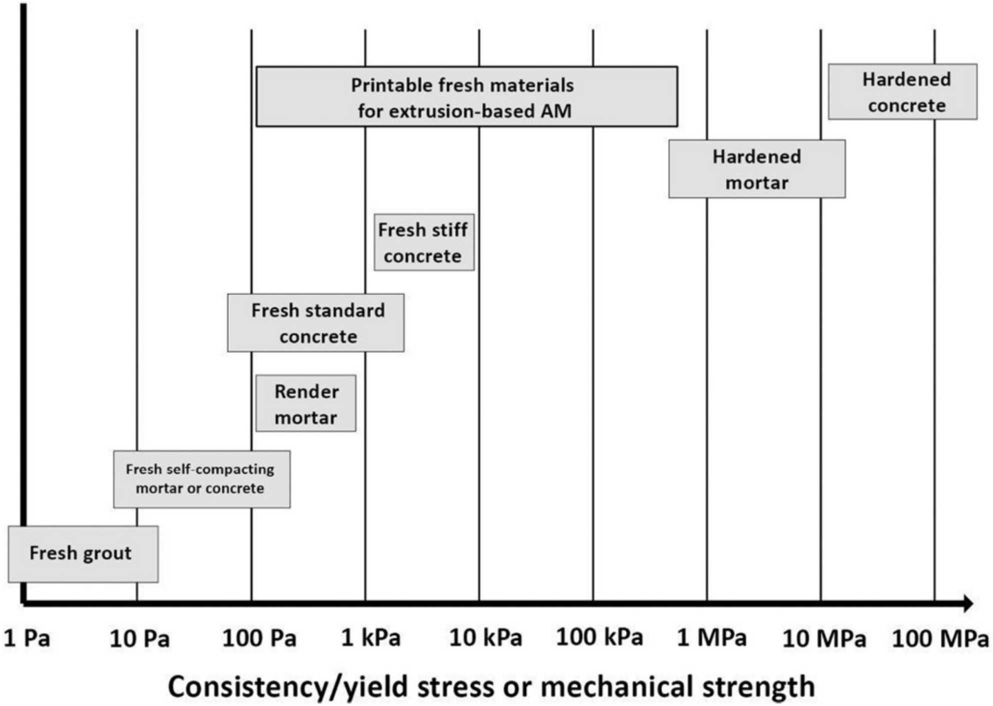
Elastic buckling



C. Matthäus

- Failure modes: **collapse, buckling, (cold Joints)**
- Models need **Yield Parameters**

➢ Need yield parameters + structural evolution for print success


Printable fresh materials for extrusion-based AM
Hardened concrete
Hardened mortar
Fresh stiff concrete
Fresh standard concrete
Render mortar
Fresh self-compacting mortar or concrete
Fresh grout
1 Pa
10 Pa
100 Pa
1 kPa
10 kPa
100 kPa
1 MPa
10 MPa
100 MPa
Consistency/yield stress or mechanical strength