

# Creation and evaluation of measured values in rheology

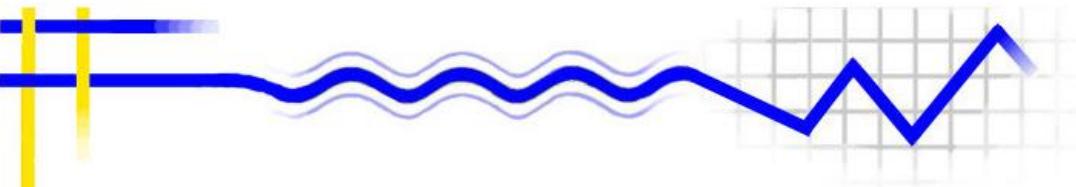


Lothar Gehm

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## Creation of measured values in rheology

$$\eta = \frac{\tau}{\dot{\gamma}} = \frac{M}{n} \frac{K_\tau}{K_{\dot{\gamma}}}$$

$\eta$	dynamic viscosity
$\tau$	shear stress
$\dot{\gamma}$	shear rate
M	torque
n	rotational speed
$K_\tau$ $K_{\dot{\gamma}}$	constants

rheometer types

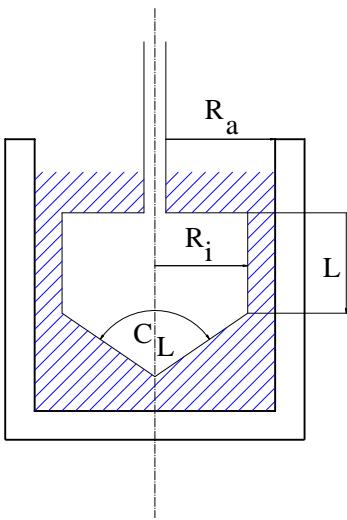
shear stress controlled CS  
shear rate controlled CR



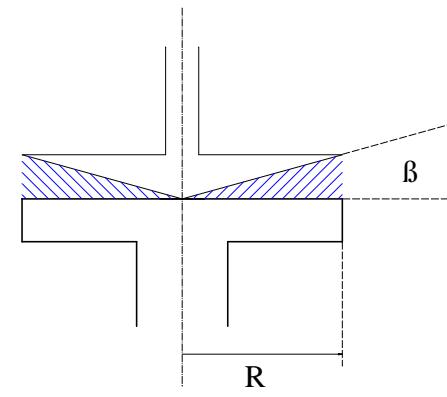
different measurement geometries

$$\eta = \frac{\tau}{\dot{\gamma}} = \frac{M}{n} \frac{K_\tau}{K_{\dot{\gamma}}}$$

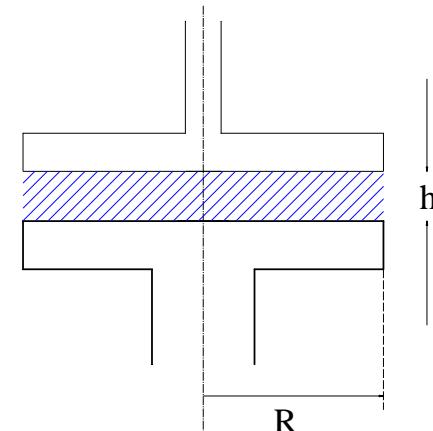
Calculation of the constants



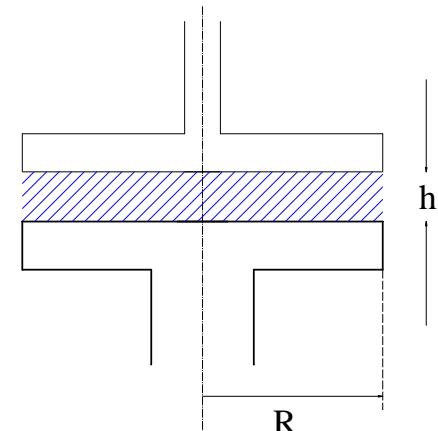
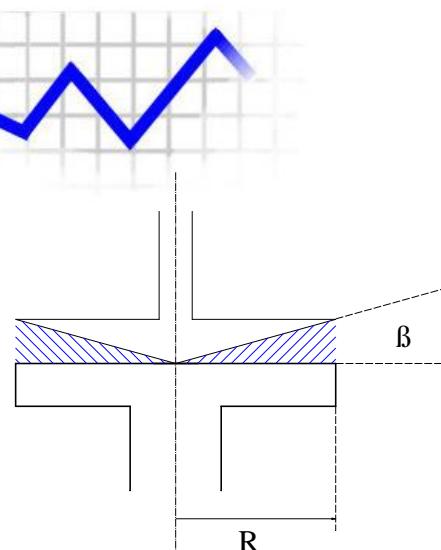
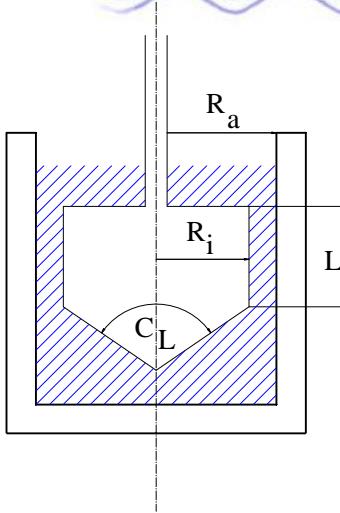
CC System



CP System



PP System



$$\tau_{(rep)} = \left( \frac{1+\delta^2}{2 \cdot \delta^2} \cdot \frac{1}{2\pi L R_i^2 C_L} \right) \cdot M$$

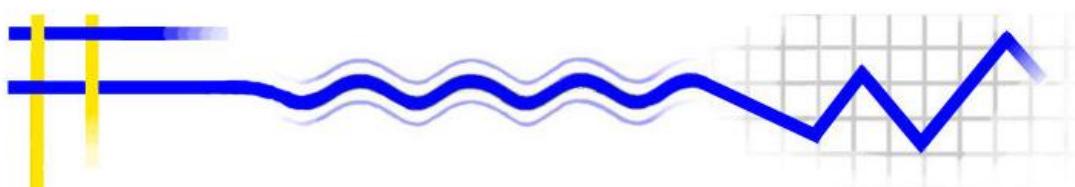
$$\dot{\gamma}_{(rep)} = \frac{1+\delta^2}{\delta^2-1} \frac{\pi}{30} n$$

$$\tau_{(r)} = \frac{3}{2\pi r^3} \cdot M$$

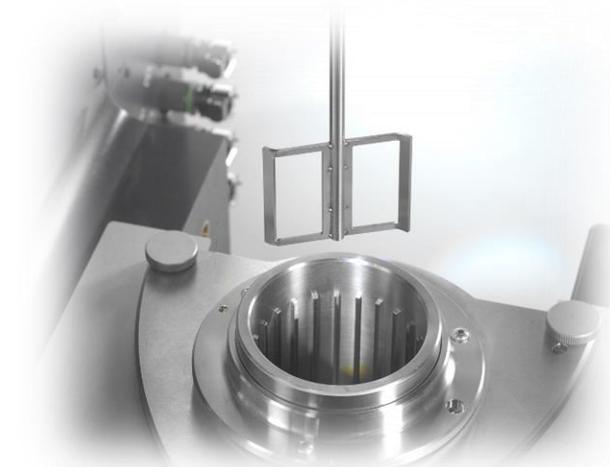
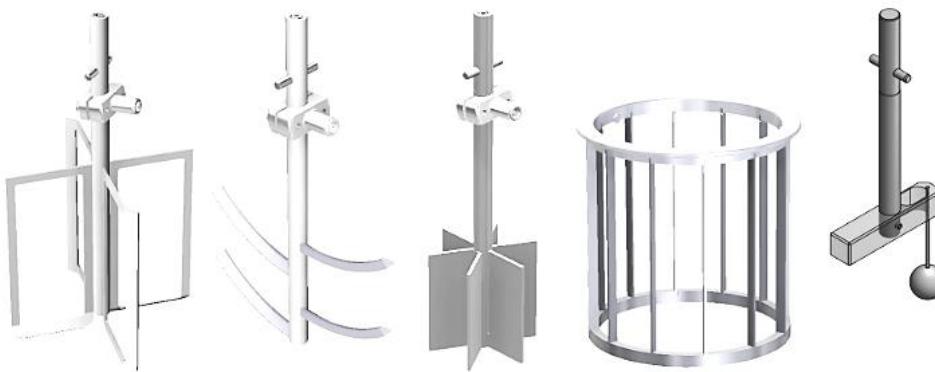
$$\dot{\gamma} = \frac{\pi}{30 \cdot \beta} \cdot n$$

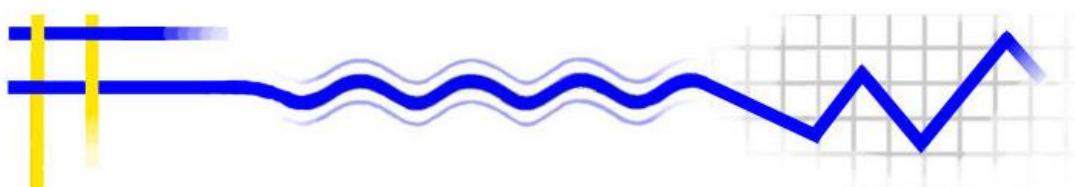
$$\tau_{(r)} = \frac{2}{\pi r^3} \cdot M$$

$$\dot{\gamma}_{(r,h)} = \frac{\pi r}{30 h} n$$



## relative measurement geometry





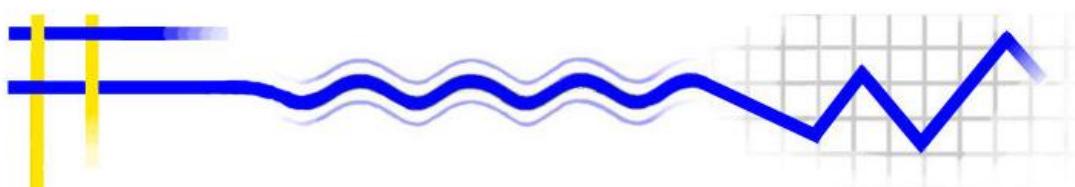
relative geometry



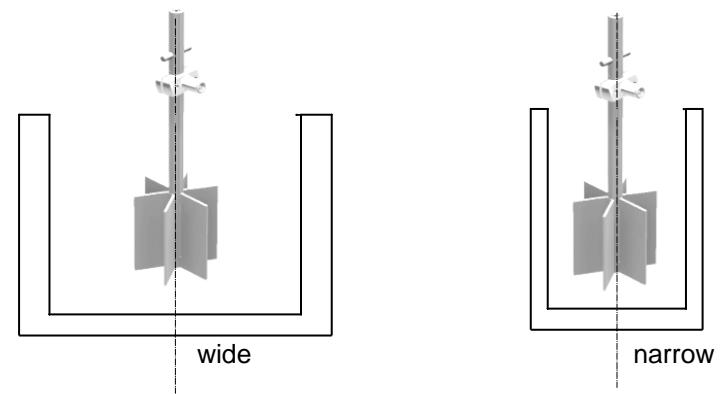
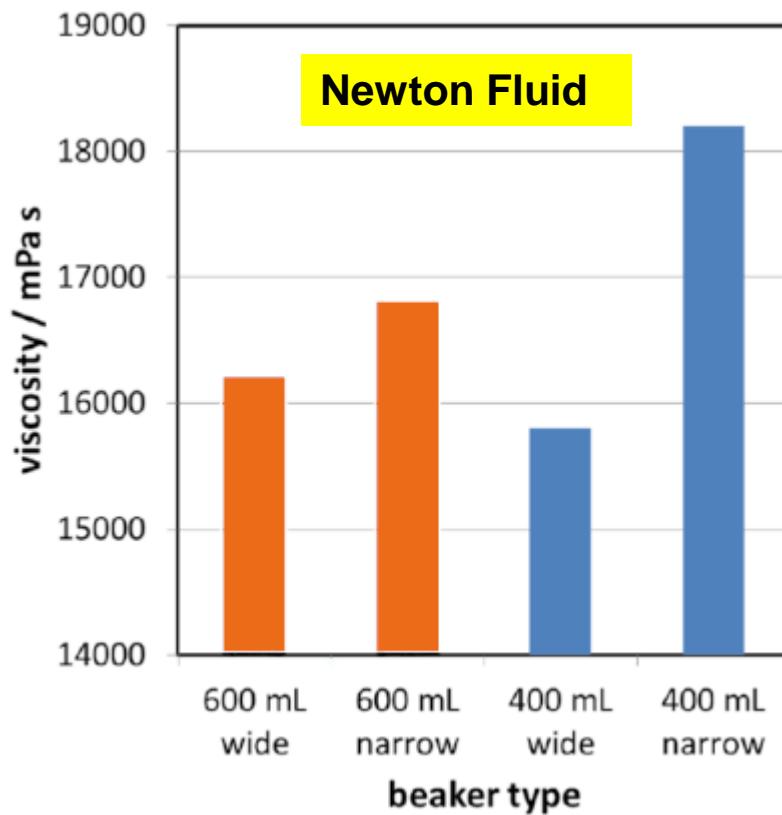
$$\tau = ? \longleftrightarrow M$$

$$\dot{\gamma} = ? \longleftrightarrow n$$

**it is not possible to calculate the constants !**

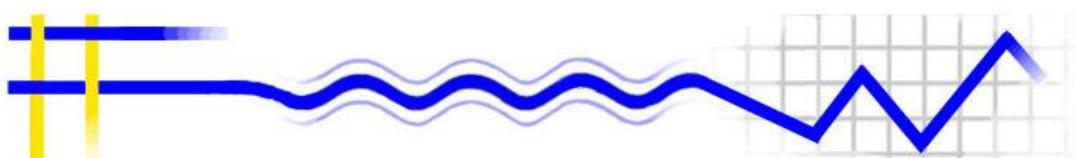


relative geometry



Versuche B. Klotz, Düsseldorf

Viscosity is a function of the measurement system - spindle and beaker



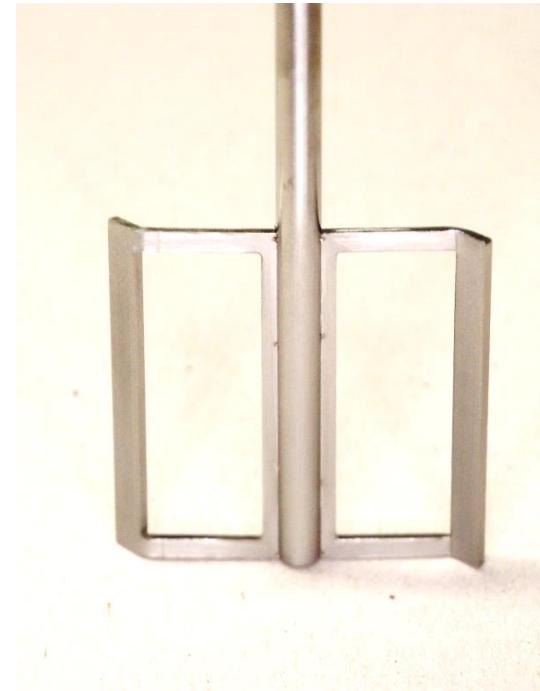
Viscosity measurement    **absolute** ./ **relative**

$$\eta = \frac{\tau}{\dot{\gamma}}$$

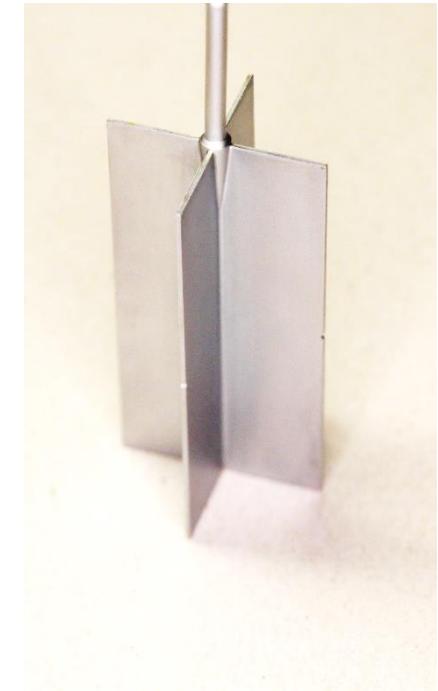
$$\eta = \frac{M}{n}$$



**absolute**  
**DIN 53019**



**PADDLE**



**relative**  
**VANE**

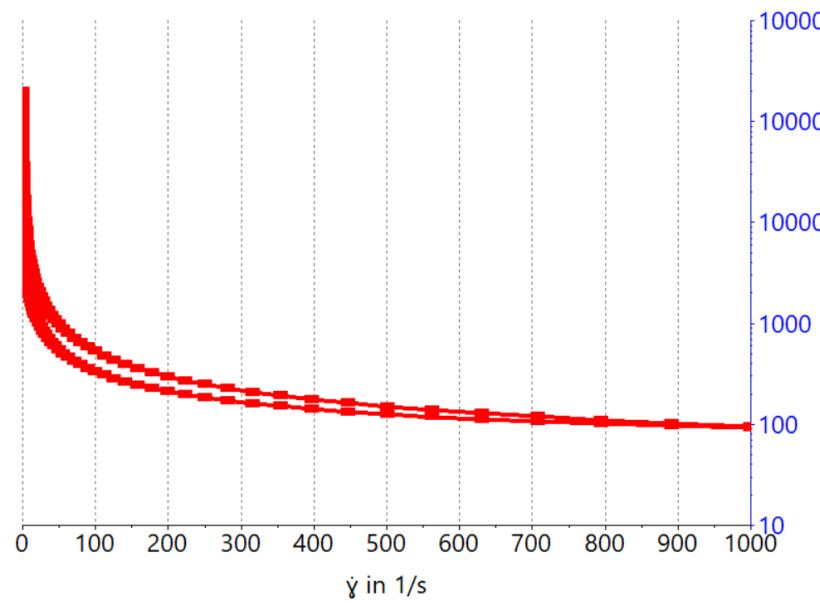
Viscosity measurement

**absolute ./ relative**

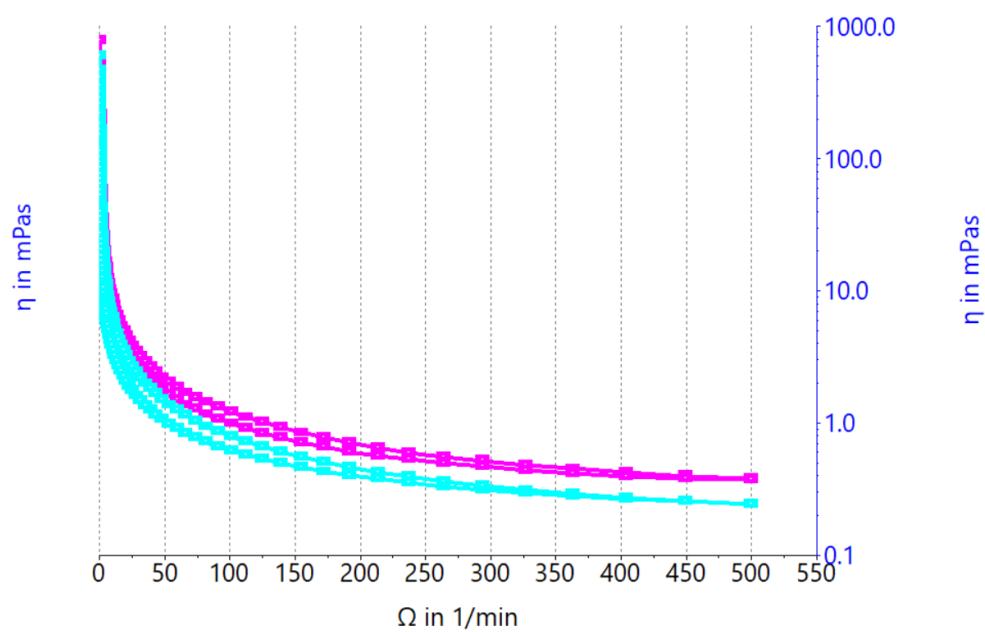
viscosity curve

$$\eta = \frac{\tau}{\dot{\gamma}}$$

$$\eta = \frac{M}{n}$$



**absolute**  
DIN 53019



**relative**  
PADDLE  
VANE

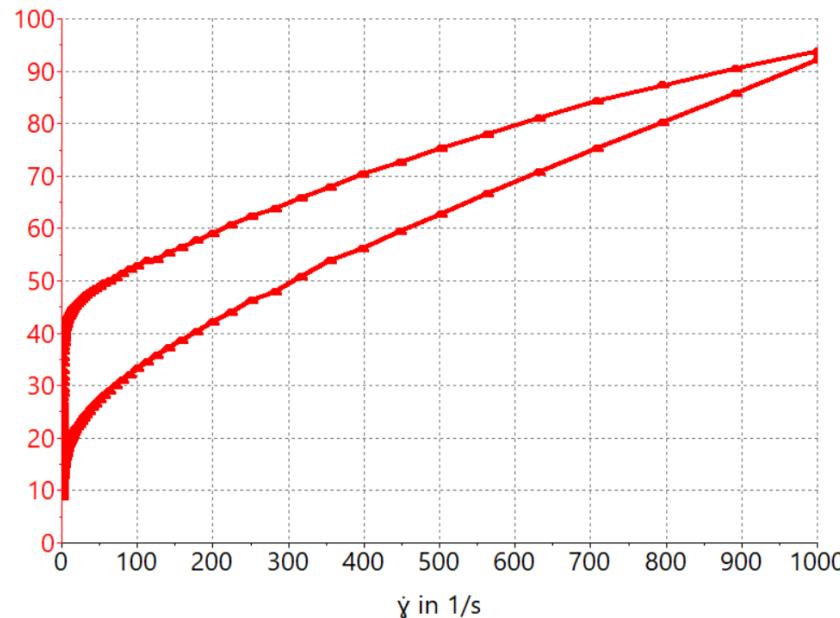
Viscosity measurement

**absolute ./ relative**

flow curve

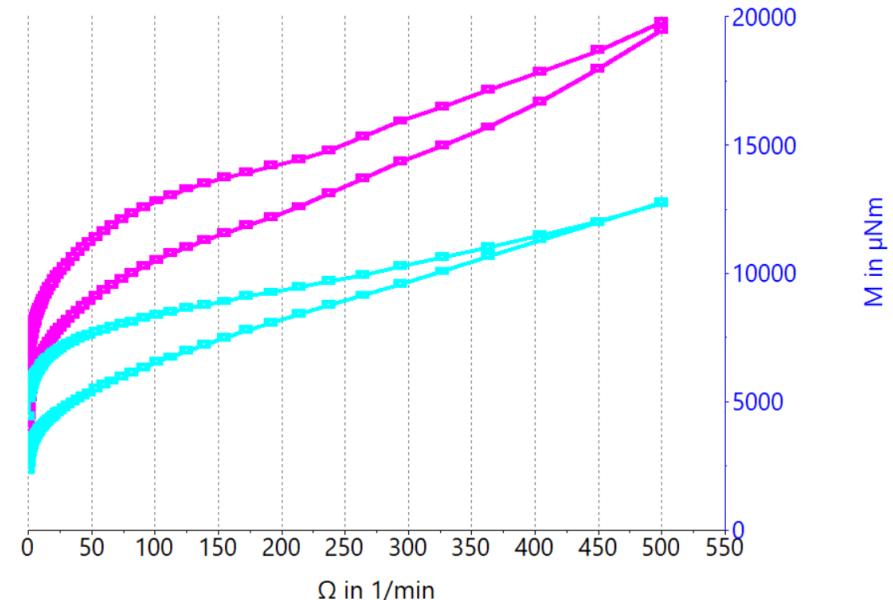
$$\eta = \frac{\tau}{\dot{\gamma}}$$

$$\eta \sim \frac{M}{n}$$



**absolute**

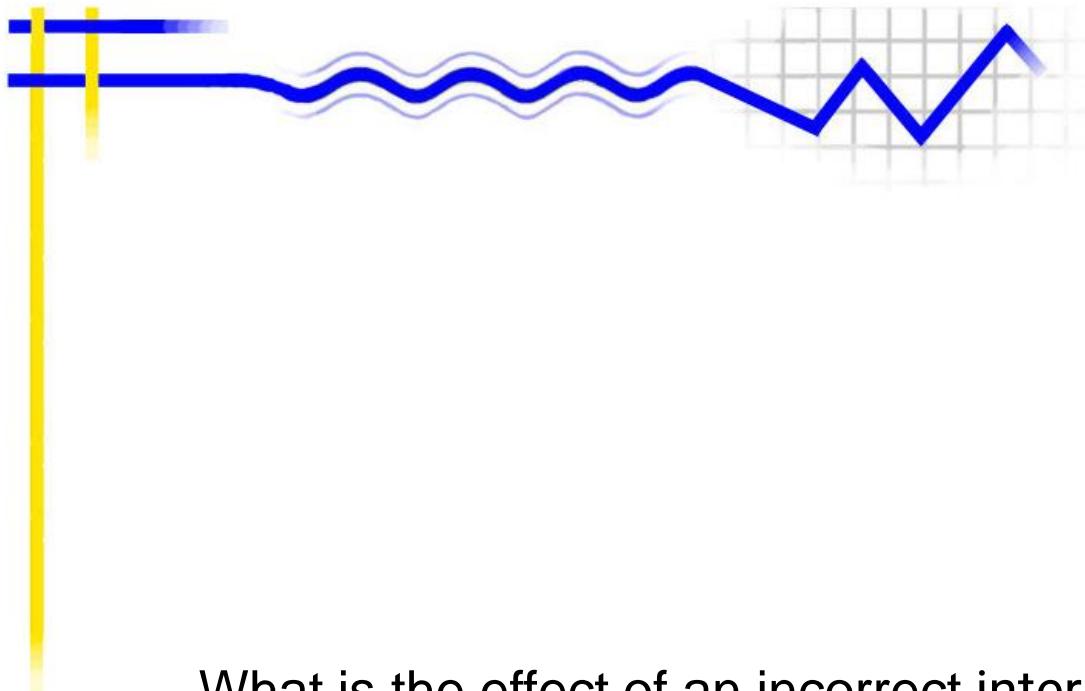
DIN 53019



**relative**

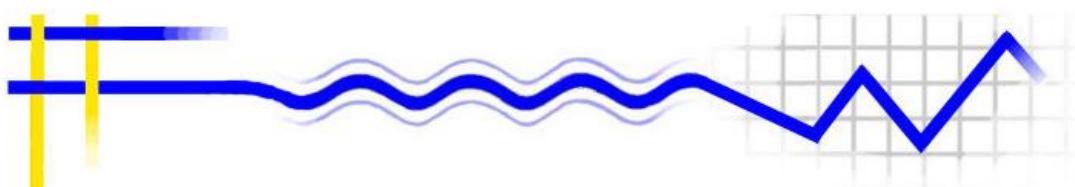
PADDLE

VANE



example

What is the effect of an incorrect interpretation of the shear stress and/or shear rate?



Pumping a non-Newtonian sample through a pipe

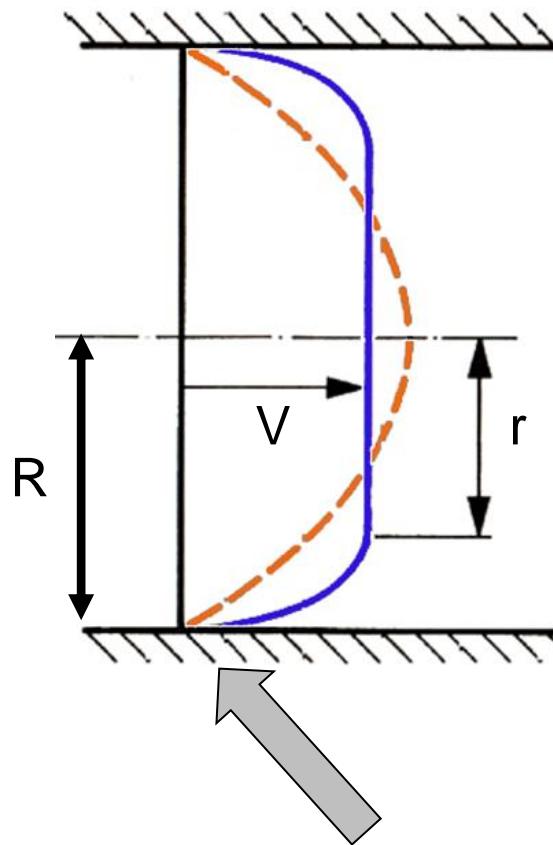
$$\eta_c = \frac{1}{\dot{\gamma}} (\sqrt{\tau} + \sqrt{\tau_{FG}})^2$$

$\eta_c$       Casson viscosity [Pa s]

$\dot{\gamma}$       Shear rate [ $s^{-1}$ ]

$\tau$       Shear stress [Pa]

$\tau_{FG}$       Casson yield point [Pa]



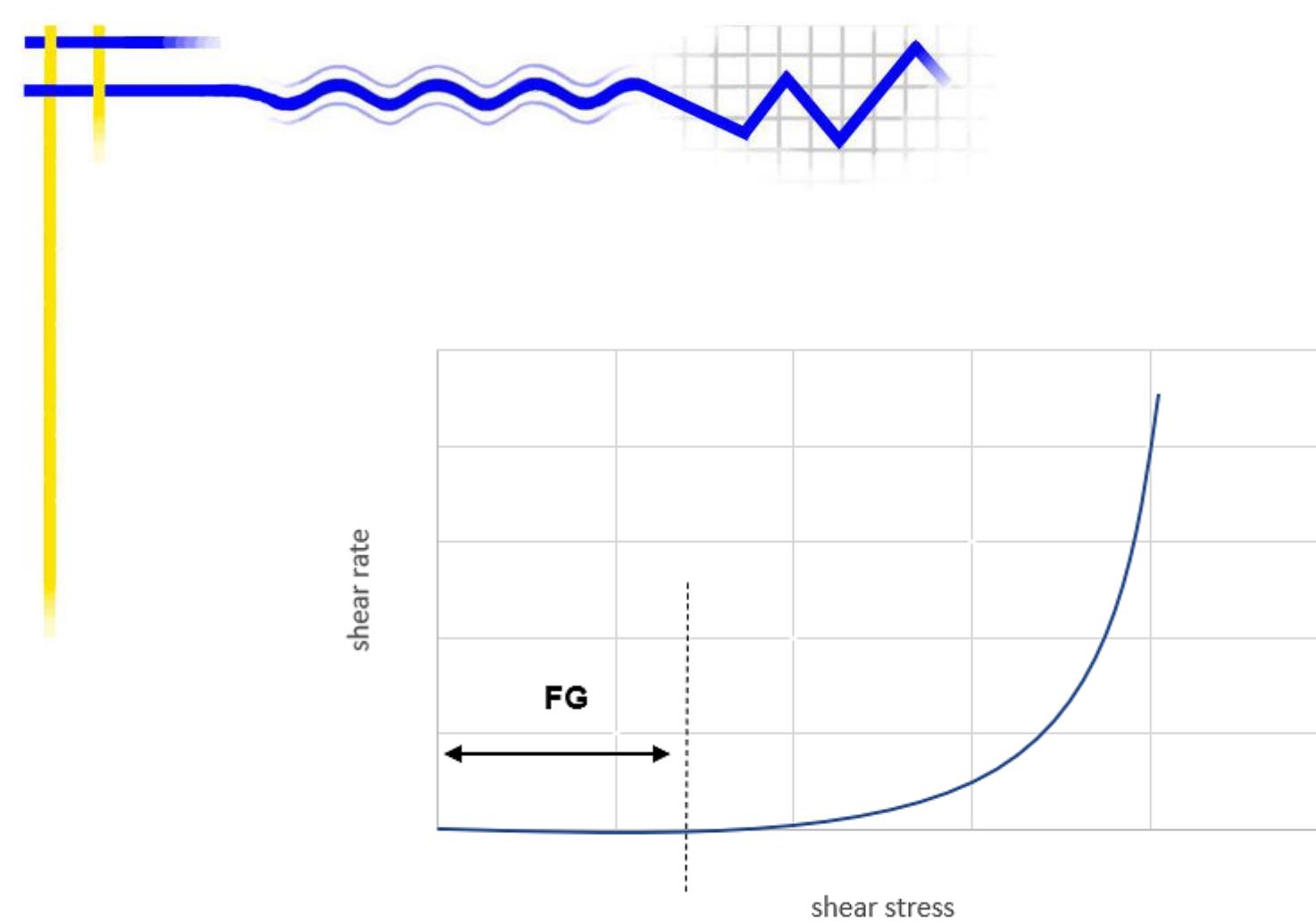
$v$  velocity

$R$  radius tube

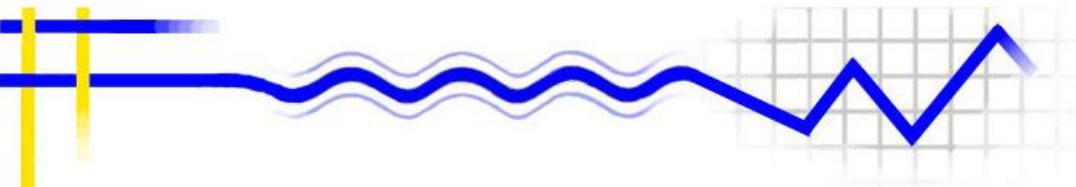
$r$  radius plug

wall adhesion and wall slip

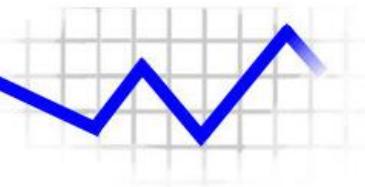
non-Newtonian sample  
newtonian sample



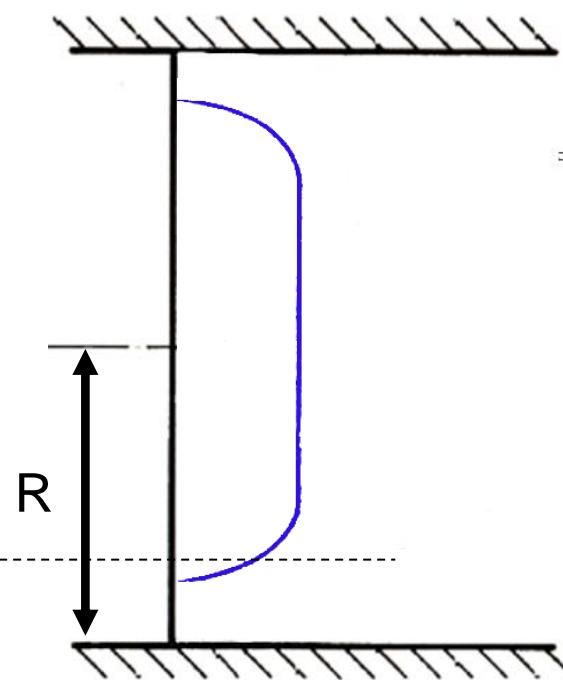
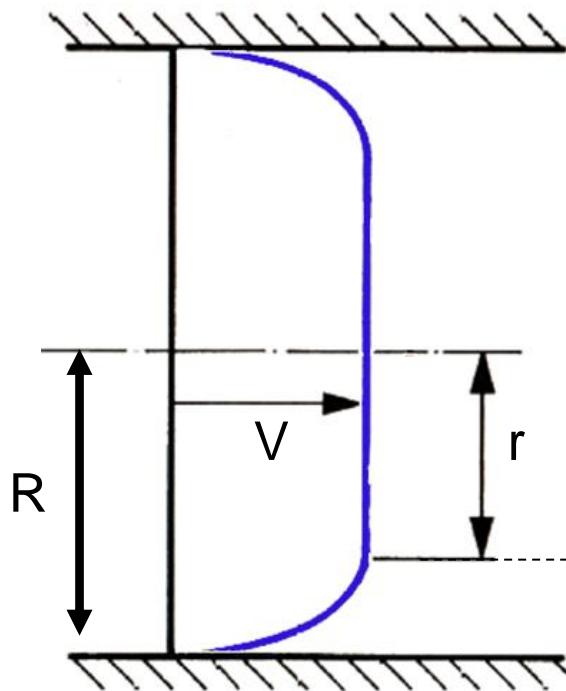
FG yield point



sample  
without yield point



sample  
with yield point



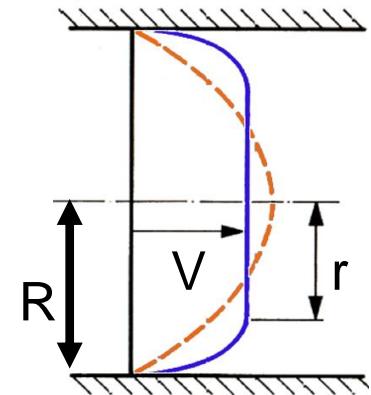
an incorrect shear stress and/or shear rate  
Fouling / Contamination caused by yield point

## Calculation of shear stress and shear rate

$$\eta_c = \frac{1}{\dot{\gamma}} (\sqrt{\tau} + \sqrt{\tau_{FG}})^2$$

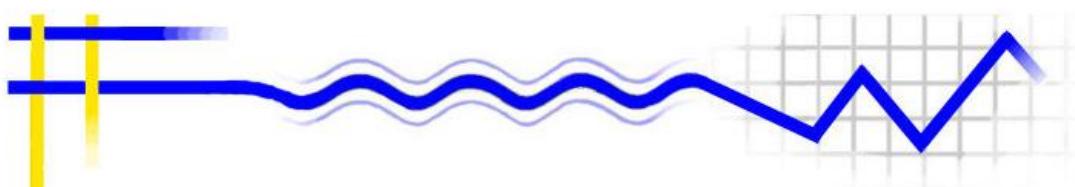
$$\tau = \frac{\Delta p}{2L} r \quad \text{shear stress}$$

$$\dot{\gamma} = \frac{1}{\eta_c} (\tau + \tau_{FG} - 2\sqrt{\tau}\sqrt{\tau_{FG}}) \quad \text{shear rate}$$



Approach for the flow rate Q

$$\frac{Q}{\pi R^3} = \frac{1}{\tau_W^3} \int_0^{\tau_W} r^2 \tau \, d\tau$$



Approach for the flow rate  $Q$

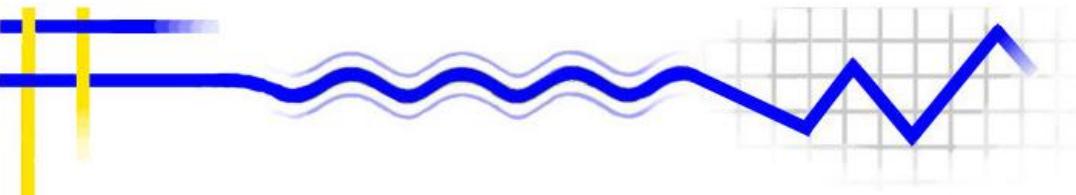
$$\frac{Q}{\pi R^3} = \frac{1}{\tau_W^3} \int_0^{\tau_w} r^2 \tau d\tau$$

$$\frac{Q \eta_c}{\pi \tau_W R^3} = \frac{1}{4} - \frac{4}{7} \sqrt{Z} + \frac{Z}{3} - \frac{z^2}{84}$$

$$Z = \frac{FG}{\tau_W}$$

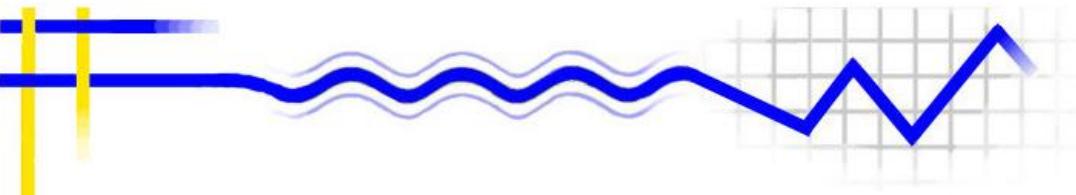
$Z=0$  corresponds to Newtonian flow,  
 $Z=1$  means pure plug flow

comparative measurements with pressure measurements on the pipe



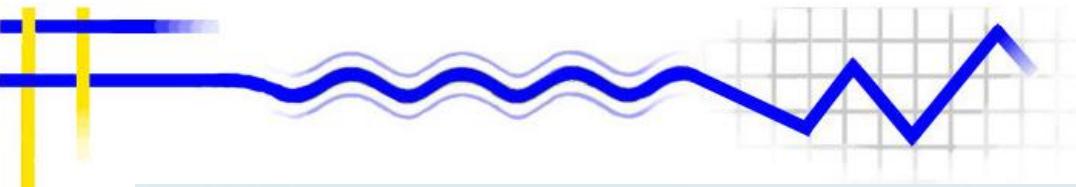
The transfer of factors found through measured with a standard liquid is not sufficient for the interpretation of shear rate and shear stress.

The viscosity cannot be determined with relative measuring systems - vane, paddle, ... - in this way.



The determination of the effectively acting shear stresses and shear rates at the application site requires sufficient experience in the interpretation of the flow curves obtained.

Approximate shear rate and shear stress can only be calculated with additional information from the system itself.



Thanks for  
your attention

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